Lissajous Curves, e.g. the Prime Knot 7_4 [*]

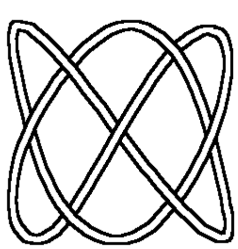
Lissajous curves are a popular family of planar curves, resp. space curves. They are complicated enough to be interesting, but regular enough to be esthetically pleasing. They are described by simple formulas:

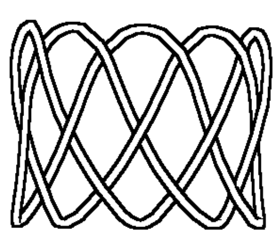
$$x(t) := aa \cdot \sin(2\pi \cdot dd \cdot t)$$
$$y(t) := bb \cdot \sin(2\pi \cdot ee \cdot t + gg)$$
$$z(t) := aa \cdot \sin(2\pi \cdot ff \cdot t + cc)$$

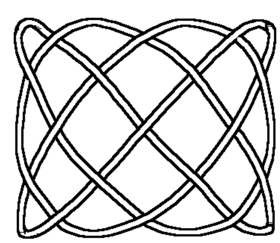
In 3DXM the parameters dd, ee, ff are rounded to integers so that the curves are closed on the interval $[0, \pi]$. The default morph varies the phase gg from 0 to $\pi/2$. – The Lissajous curves are also physically interesting, they describe the joint motion of orthogonal uncoupled oscillators $(x(t), y(t), z(t))$ with different frequencies.

A **prime knot** is not the knot sum of smaller knots. For example, Square Knot and Granny Knot are not prime: each is a sum of two Trefoil Knots. There are 14 prime knots with the minimal number of crossings at most 7, see the documentation ("About This Object" or ATO) for "V. Jones Braid List". The 4th 7-crossings-knot, the prime knot 7_4, is our default Lissajous space curve, $(dd, ee, ff, gg) = (2, 3, 7, \pi/2)$. – Two other alternating examples are:

$$(dd, ee, ff) = (2, 5, 13) \text{ resp. } = (4, 3, 23).$$







[*]This file is from the 3D-XploreMath project.
Please see http://www.math.uci.edu/~palais/ or http://3d-xplormath.org/

There are 249 prime knots with at most 10 minimal number of crossings. One can visualize those via the Space Curves Menu entry: "V. Jones Braid List". The notion of prime knot is important because Horst Schubert proved that the decomposition of a knot as knot sum (= connected sum) of prime knots is unique. The knot invariants are a good way to check whether a given knot is a prime knot.

There is an easy sufficient criterion that guarantees that the knot under consideration cannot be drawn with fewer crossings: The thread of the knot has to pass alternatingly through overcrossings and undercrossings. Such knots are called **alternating knots**. Alternating knots are always non-trivial. – There is no similarly simple criterion to recognize a knot as prime.

H.K.